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Anticommutative Electric and Magnetic Charges, and the Monopole Question

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The monopole question is treated anew in the light of a recent, strictly covariant, extended formulation of fermion quantum field theory naturally including also a pseudoscalar variety of conserved charges. The essential novelty lies in the resulting quantum property of anticommutivity between scalar and pseudoscalar charge varieties, which should in particular apply to electric and magnetic charges. As an immediate outcome, there should no longer be any (Dirac-like) quantization condition relating these charges and binding the magnetic elementary charge to have a very great strength. A generalized Lagrangian approach to the monopole problem is made truly viable, leading to two independent local gauge couplings which are separately generated by the electric and magnetic elementary charges and are not allowed to interfere. This would prevent electric and magnetic monopoles from mutually interacting and would particularly account for the "absence" of magnetic sources in ordinary electromagnetism. Within such a framework, an electric charge eigenstate with a nonzero eigenvalue is bound to have a null magnetic charge expectation value, and the magnetic dipole moment of an electrically charged point fermion may actually be seen as resulting from the additional internal presence of a single magnetic charge subjected to a maximal uncertainty in sign. An easy estimate makes it allowable to assign to this charge a strength just equal to that of the partner electric charge. Such a conjecture leads to a "dual" model of a charged point fermion where the "electric" and "magnetic" roles can well be interchanged with no observable effects. In the associated formalism, duality symmetry is already included without the need to appeal to any "missing" electromagnetic phenomenology to be discovered.

1. INTRODUCTION

The question of monopoles has been an intriguing subject of theoretical investigation for nearly 70 years. Most works concerned with it are essentially developments of Dirac's fundamental papers [1]. Two main monopole models

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have emerged during the last 40 years. These are the 't Hooft–Polyakov monopole (which corresponds to a soliton solution within a scheme of spontaneously broken weak-isospin symmetry) [2–5] and the Wu–Yang monopole (which relies upon the fiber-bundle theory) [6–8]. Both models are able to avoid the Dirac "string" on dealing with the motion of an electric charge in the field produced by a magnetic monopole. Neither of them, however, can get rid of two basic difficulties arising from Dirac's theory:

(i) Due to the Dirac quantization condition (in Heaviside–Lorentz units) $qg = 2\pi n$ ($n = 1, 2, \ldots; \hbar = c = 1$), the magnetic elementary charge g_0 and the electric one q_0 should be related, $g_0 = 2\pi/q_0$, in such a way that the coupling constant $g_0{}^2/4\pi = (1/4)(4\pi/q_0{}^2)$ is nearly 5000 greater than $q_0{}^2/4\pi$! This, in view of electroweak unification, seems to be a rather embarassing outcome, as the couplings generated by electric and magnetic elementary charges should all the more reason be expected to be two mere manifestations of a *single* interaction.

(ii) Even though some noteworthy attempts have been made [10, 13], a *strictly orthodox* Lagrangian formulation appears to be no longer allowable when magnetic monopoles are included.

The origin of these two questionable points lies indeed in the conjecture, seemingly obvious, at first sight, of a *mutual* interaction for electric and magnetic monopoles: the whole electromagnetic field that can be felt by one kind of monopole should generally include the field contribution coming from the other kind of monopole. Here it will be argued, nevertheless, that such an interaction cannot be taken for granted at all due to the *pseudoscalar* (rather than scalar) behavior to be expected for a single magnetic charge [11–18]. This can be properly seen within a recent generalized approach to fermion quantum field theory naturally including also a pseudoscalar variety of conserved charges [18]. The essential point is that the latter *anticommutes* with the ordinary (scalar) charge variety, so that a fermion bearing both (scalar and pseudoscalar) kinds of conserved charges might display only *one* charge variety at a time, with a *null* expectation value for all charges of the other kind (see Section 2).

On these grounds, a pure *quantum* view of the monopole problem can be developed (see Sections 3 and 4). It is able to give an immediate understanding of the fact that *magnetic four-currents locally coexisting with electric four-currents* have never been observed: due to the anticommutivity property between scalar and pseudoscalar charge varieties, electric-charge eigenstates are now strictly bound to have a *null* magnetic-charge expectation value (unless their electric charge is vanishing). Both points (i) and (ii) can furthermore be overcome by the new approach since a *strict* Lagrangian formulation (with no singularity problems) is actually made viable where the electric and magnetic elementary charges may in principle generate only *two separate* *local gauge couplings unable to interfere*: an electric (magnetic) monopole can now couple merely to the four-potential originated by electric (magnetic) currents. Of course, dealing with a pair of distinct (electric and magnetic) four-potentials is not really a novelty in the literature [9, 10, 13, 16], but in the present case they also stand for two *independent* (vector and axial-vector) gauge fields. All this fully accounts for the empirical “absence” of magnetic sources in electromagnetism where electric charges are involved.

In light of this approach, any existing charged point fermion may be thought of anew (see Section 5): its apparent “magnetic dipole” behavior would be actually generated by the additional internal presence of *just a single magnetic point charge subjected to a maximal uncertainty in sign*. From an easy estimate, one can further see that the magnetic charge in question may even be assumed to have the *same* strength as the partner electric charge. Such an assumption strictly involves a “dual” model of a charged point fermion (see Section 6), where the antithetical roles played by the two coexisting (electric and magnetic) charges can be interchanged (by a duality transformation) *without any observable effects*. The associated formalism is already so structured as to possess duality symmetry, with no need to bring in some “missing” electromagnetic phenomenology which is still to be discovered as a “complement” of the ordinary one. In view of such a model, one might conclude that if matter is really made up of spin-1/2 point fermions only, it would be wrong to think of magnetic monopoles *as new objects yet undiscovered*.

1. UNIFIED, STRICTLY COVARIANT, FERMION–ANTIFERMION QUANTUM FIELD FORMALISM, AND NATURAL APPEARANCE OF A PSEUDOSCALAR CHARGE VARIETY WITH NO BREAKING OF PARITY SYMMETRY

In this section as well as in the next one I review some relevant points of the generalized (strictly covariant) approach to fermion quantum field theory which has been outlined in refs. 18.

The usual fermion quantum field formalism, based on the Dirac equation and the “hole” interpretation for negative frequencies, is obviously unable to provide a one-particle relativistic description: it deals with a Fock space as the sum of two distinct (and not fully covariant) pure Fock spaces—relevant to (positive-energy) “particles” and “holes,” respectively—which are mapped onto each other by charge conjugation, or “particle” $\rightleftarrows$ “hole” conjugation. A fully covariant, one-particle description is made viable using the Stüeckelberg–Feynman improved approach to the negative-energy problem [19]: the motion of a “hole” can then be reinterpreted as a motion *backward* in time of a negative-energy "particle," and the composite Fock space can accordingly be recast as a *single* Fock space for "particles" only, with energies now covariantly running over the *whole* spectrum of positive and negative eigenvalues. Let us denote the latter (fully covariant) Fock space by $\mathcal{F}^{o}$. In line with this, the Stüeckelberg–Feynman approach also enables one to think of a *covariant* charge conjugation which may *globally* interchange either a positive- or negative-energy fermion and either a positive- or negative-energy antifermion: in principle, this is *not* just the same as "particle" $\rightleftarrows$ "hole" conjugation (which is a noncovariant operation interchanging positive-energy fermions and antifermions). There is, however, one apparent difficulty. The single Fock space $\mathcal{F}^{o}$ can *equally* pertain (covariantly) to either identical fermions or identical antifermions: according to the Stüeckelberg–Feynman view, a complete set of $\mathcal{F}^{o}$ kets (bras) for fermions amounts to a complete set of $\mathcal{F}^{o}$ bras (kets) for antifermions. Such a difficulty can be overcome by carefully reexamining the Stüeckelberg–Feynman approach in classical relativistic terms. Let $-p^{\mu} = m(-u^{\mu})$ ($\mu = 0, 1, 2, 3$; metric: $+ - - -$) be the four-momentum of a negative-energy particle of proper (i.e., covariant) mass m (> 0) and four-velocity $-u^{\mu} = -dx^{\mu}/ds$ ($-dx^{0} < 0$). Since the equivalent positive-energy antiparticle, with four-momentum p^{μ}, is covering just the same world-line in the opposite direction, $ds \rightarrow -ds$, the "slope" $-u^{\mu}$ of that world-line cannot be affected by the reinterpretative procedure: one has $(-dx^{\mu})/ds = dx^{\mu}/(-ds)$. Strictly speaking, therefore, the procedure is *such that* $-p^{\mu} \rightarrow p^{\mu} \Rightarrow m \rightarrow -m$. On these theoretical grounds, one may state that a Dirac fermion and a Dirac antifermion can *covariantly* be distinguished by the (opposite) sign of their proper mass; so that the covariant charge conjugation we are looking for is to be identified with *proper-mass conjugation* [20–23]. As $\mathcal{F}^{o}$ can equally refer to either fermions or antifermions (with both positive and negative energies), we must expect it to be left invariant by proper-mass conjugation: this corresponds to the fact that the proper-mass sign in the Dirac equation is irrelevant. To get really a *non*trivial definition of a covariant charge conjugation, one should therefore *double* $\mathcal{F}^{o}$ by giving it some "label" that may specifically tell which of the two proper-mass signs is being considered. For this purpose it is appropriate to introduce two (orthogonal) unit internal state vectors $|f\rangle$ and $|\bar{f}\rangle$ which are eigenvectors of a (covariant) one-particle proper-mass operator M with eigenvalues $+m$ and $-m$:

$$M|f\rangle = +m|f\rangle, \qquad M|\bar{f}\rangle = -m|\bar{f}\rangle \tag{2.1}$$

Let $\mathcal{S}_{\text{in}}$ be the two-dimensional internal space that is spanned by such eigenvectors. A "dressed" generalized Fock space $\mathcal{F}$ can then be built from the "bare" one $\mathcal{F}^{o}$ such that

$$\mathcal{F} \equiv \mathcal{F}^{o} \otimes \mathcal{S}_{in} \tag{2.2}$$

In this way, the complete set of $\mathcal{F}^{o}$ kets (bras) may undergo a *doubling* into a "Dirac fermionic" set covariantly labeled by $|f\rangle$ ($\langle f|$) plus a "Dirac antifermionic" one covariantly labeled by $|\bar{f}\rangle$ ($\langle\bar{f}|$) (with an energy range, in either case, still including both positive and negative eigenvalues); moreover, the covariant charge conjugation may be represented by a unitary and Hermitian operator C essentially acting in $\mathcal{S}_{in}$ and anticommuting with M:

$$C|f\rangle = |\bar{f}\rangle, \qquad C|\bar{f}\rangle = |f\rangle \qquad (C^{-1} = C^{\dagger} = C) \tag{2.3}$$

What such a doubling involves can be fully understood on coming back to the "particle–hole" language: it provides two *alternative* (equivalent) Dirac pictures where one is unambiguously choosing *either* "particle" = fermion and "hole" = antifermion *or* "particle" = antifermion and "hole" = fermion, respectively. These are two *proper-mass conjugated* descriptions, to be associated with two opposite-proper-mass Dirac free-field equations like

$$i\gamma^{\mu}\partial_{\mu}\psi_{f} = +m\psi_{f}, \qquad i\gamma^{\mu}\partial_{\mu}\psi_{\bar{f}} = -m\psi_{\bar{f}} \tag{2.4}$$

($\hbar = c = 1$; $\gamma^{0\dagger} = \gamma^{0}$, $\gamma^{k\dagger} = -\gamma^{k}$, $k = 1, 2, 3$) where $\psi_{\bar{f}}$ should consistently stand for the *proper-mass conjugated* counterpart of ψ_{f}. Both field equations (2.4) are equally allowable within $\mathcal{F}^{o}$, and the dressed Fock space (2.2) should go along with a *double*-structured, dressed field operator of the type

$$\Psi(x) = \psi_{f}(x)\langle f| + \psi_{\bar{f}}(x)\langle\bar{f}| \tag{2.5}$$

($x \equiv x^{\mu}$). This is a Lorentz four-spinor, also looking like an $\mathcal{S}_{in}$ (bra) vector of "Dirac components" $\psi_{f}(x)$ and $\psi_{\bar{f}}(x)$ (whose orthogonality in $\mathcal{S}_{in}$ is just ensured by their being two proper-mass eigenfields with different eigenvalues). The field component $\psi_{f}(x)\langle f|$ can covariantly annihilate (either positive- or negative-energy) *Dirac fermions*, and the same holds for $\psi_{\bar{f}}(x)\langle\bar{f}|$ as regards (either positive- or negative-energy) *Dirac antifermions*. According to (2.3), the C-conjugate field operator reads

$$\Psi^{(C)}(x) \equiv \Psi(x)C = \psi_{f}(x)\langle\bar{f}| + \psi_{\bar{f}}(x)\langle f| \tag{2.6}$$

and a glance at (2.6) shows that applying C may equivalently be seen as putting

$$C\colon \quad \psi_{f}(x) \rightleftarrows \psi_{\bar{f}}(x) \tag{2.7}$$

Hence it follows that $\psi_{\bar{f}}(x)$ should be *covariantly* obtained from $\psi_{f}(x)$ (up to a phase factor) by applying proper-mass reversal to the Dirac equation:

$$\psi_{\bar{f}}(x) = \gamma^{5}\psi_{f}(x), \qquad \overline{\psi}_{\bar{f}}(x) = -\overline{\psi}_{f}(x)\gamma^{5} \tag{2.8}$$

($\overline{\psi} = \psi^{\dagger}\gamma^{0}$; $\gamma^{5} \equiv i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}$). In line with (2.8) (and with the fact that C is defined in $\mathcal{S}_{in}$) the standard Fourier expansions of $\psi_{f}(x)$ and $\psi_{\bar{f}}(x)$ must contain *identical* "particle"-annihilation operators of the type $a(\mathbf{p}, \sigma)$ as well as *identical* "hole"-creation operators of the type $a^{h\dagger}(\mathbf{p}, \sigma)$ (σ being the helicity variable). This is admissible because $\psi_f(x)$ and $\psi_{\bar{f}}(x)$ belong to *alternative* (proper-mass conjugated) Dirac pictures where one has either "particle" = fermion (and "hole" = antifermion) or "particle" = antifermion (and "hole" = fermion), each single picture being able (independently of the other) to account for the creation or annihilation of a "particle"–"hole" pair. Here $\psi_{\bar{f}}(x)$ has nothing to do with the customary "hole" field which is (noncovariantly) obtained by applying "particle" ⇄ "hole" conjugation (i.e., $a \to a^h$, $a^{h\dagger} \to a^\dagger$): the latter can still be encountered, too, but only *within* either picture, as a result of normal ordering. By exploiting Eqs. (2.8) and introducing the adjoint field operator

$$\overline{\Psi}(x) = |f\rangle\overline{\psi}_f(x) + |\bar{f}\rangle\overline{\psi}_{\bar{f}}(x) \tag{2.9}$$

one can compactly write

$$\Psi^{(C)}(x) = \gamma^5\Psi(x), \qquad \overline{\Psi}^{(C)}(x) = -\overline{\Psi}(x)\gamma^5 \tag{2.10}$$

This strictly defines the effective action of chirality γ^5 as covariant charge conjugation.

Another relevant $\mathcal{S}_{\text{in}}$ basis can be obtained from $(|f\rangle, |\bar{f}\rangle)$ by performing the rotation

$$\begin{aligned} |f\rangle &= 2^{-1/2}(|f^{\text{ch}}\rangle + |\bar{f}^{\text{ch}}\rangle) \\ |\bar{f}\rangle &= 2^{-1/2}(-|f^{\text{ch}}\rangle + |\bar{f}^{\text{ch}}\rangle) \end{aligned} \tag{2.11}$$

The operator C is made diagonal in such a basis:

$$C|f^{\text{ch}}\rangle = -|f^{\text{ch}}\rangle, \qquad C|\bar{f}^{\text{ch}}\rangle = |\bar{f}^{\text{ch}}\rangle \tag{2.12}$$

Thus a further (unitary and Hermitian) operator, say P_{in}, can be introduced in $\mathcal{S}_{\text{in}}$, which in turn has the property of interchanging $|f^{\text{ch}}\rangle$ and $|\bar{f}^{\text{ch}}\rangle$,

$$P_{\text{in}}|f^{\text{ch}}\rangle = |\bar{f}^{\text{ch}}\rangle, \qquad P_{\text{in}}|\bar{f}^{\text{ch}}\rangle = |f^{\text{ch}}\rangle \qquad (P_{\text{in}}^{-1} = P_{\text{in}}^\dagger = P_{\text{in}}) \tag{2.13}$$

and being diagonal in the basis $(|f\rangle, |\bar{f}\rangle)$. More specifically, since

$$P_{\text{in}}|f\rangle = |f\rangle, \qquad P_{\text{in}}|\bar{f}\rangle = -|\bar{f}\rangle \tag{2.14}$$

one has that P_{in} may be regarded (apart from a phase constant $\eta = \pm 1$) as standing for an "intrinsic parity" covariant operator: it should be identified with that factor of the parity operator P ($\equiv P_{\text{ex}}\, P_{\text{in}} = P_{\text{in}}\, P_{\text{ex}}$) which properly acts in $\mathcal{S}_{\text{in}}$ (the other factor, P_{ex}, properly acting in $\mathcal{F}^o$). In the new $\mathcal{S}_{\text{in}}$ basis, the field $\Psi(x)$ reads

$$\Psi(x) = \chi_f(x)\langle f^{\text{ch}}| + \chi_{\bar{f}}(x)\langle \bar{f}^{\text{ch}}| \tag{2.15}$$

where

$$\chi_f(x) \equiv 2^{-1/2}(1 - \gamma^5)\psi_f(x), \qquad \chi_{\bar{f}}(x) \equiv 2^{-1/2}(1 + \gamma^5)\psi_{\bar{f}}(x) \quad (2.16)$$

and

$$\psi_f = 2^{-1/2}(\chi_f + \chi_{\bar{f}}), \qquad \psi_{\bar{f}} = 2^{-1/2}(-\chi_f + \chi_{\bar{f}}) \quad (2.17)$$

In this way, one can naturally obtain the two (massive) *chiral* fields χ_f and $\chi_{\bar{f}}$, with *opposite* chiralities, on the same footing as the two Dirac fields ψ_f and $\psi_{\bar{f}}$. Such an outcome takes on a very special meaning in the zero-mass limiting case: for $|m| = 0$, in line with experience, the strictly covariant quantum approach here considered leads to an *automatic* prediction of an *only left-handed* chiral fermion and an *only right-handed* chiral antifermion, which are now to be reinterpreted as two C eigenstates being nothing but *ordinary* mirror images of each other. This should be connected with the more general fact that, as is discussed in refs. 8, the new field formalism in hand makes legitimate a *parity-symmetric* reinterpretation of the "maximal parity violation" effect [24–31] peculiar to the weak-isospin fermionic current.

A full understanding of all this can be gained by introducing two one-particle "charge" operators Q and Q^{ch}, the former being diagonal (with opposite nonzero eigenvalues) in the "Dirac" $\mathcal{S}_{in}$ basis $(|f\rangle, |\bar{f}\rangle)$ and the latter being the same in the "chiral" $\mathcal{S}_{in}$ basis $(|f^{ch}\rangle, |\bar{f}^{ch}\rangle)$: they are such that

$$CQ = -QC, \qquad P_{in}Q = QP_{in} \quad (2.18)$$

and

$$P_{in}Q^{ch} = -Q^{ch}P_{in}, \qquad CQ^{ch} = Q^{ch}C \quad (2.19)$$

Thus Q behaves like a *scalar* charge (reversed by C) and Q^{ch} like a *pseudoscalar* charge (reversed by P_{in}); and one has that C and P_{in} properly stand for *scalar*- and *pseudoscalar-charge conjugation* operators, respectively. Hence, in view of Eqs. (2.3) and (2.13), it follows that the internal states $(|f\rangle, |\bar{f}\rangle)$ look like *pure* scalar-charge conjugated eigenstates and the internal states $(|f^{ch}\rangle, |\bar{f}^{ch}\rangle)$ like *pure* pseudoscalar-charge conjugated eigenstates. This corresponds to the fact that Q and Q^{ch} are *anticommuting* operators,

$$Q\,Q^{ch} + Q^{ch}Q = 0 \quad (2.20)$$

though their squares clearly satisfy the commutation relations

$$[Q^2, Q^{ch}] = [(Q^{ch})^2, Q] = 0 \quad (2.21)$$

Each of the two charges Q and Q^{ch}, if singly applied (from the right) to the field $\Psi(x)$, is able to *superselect* that internal representation of $\Psi(x)$, either

(2.5) or (2.15), diagonalizing it. If such *superselections* are to be ascribed a physical meaning, the two bases ($|f\rangle$, $|\bar{f}\rangle$) and ($|f^{ch}\rangle$, $|\bar{f}^{ch}\rangle$) should also give two allowable pairs of *superselected* internal states for the fermion and the antifermion. The most general situation is when the *same* fermion–antifermion pair should be described by either the Dirac or the chiral $\mathcal{S}_{in}$ basis according to whether a charge Q or Q^{ch} is involved. In such a situation, the *maximal incompatibility* between the superselections induced by Q and Q^{ch} turns out to play a crucial role: one alternately has

$$\langle f|Q^{ch}|f\rangle = \langle \bar{f}|Q^{ch}|\bar{f}\rangle = 0 \tag{2.22}$$

when ($|f\rangle$, $|\bar{f}\rangle$) is superselected, and

$$\langle f^{ch}|Q|f^{ch}\rangle = \langle \bar{f}^{ch}|Q|\bar{f}^{ch}\rangle = 0 \tag{2.23}$$

when ($|f^{ch}\rangle$, $|\bar{f}^{ch}\rangle$) is superselected [13]. Hence it may further be concluded that the "true" fermion $\rightarrow$ antifermion *covariant* conjugation should generally be identified with CP_{in}, even though CP_{in} is just reducible to C when acting on $|f\rangle$ and to P_{in} when acting on $|f^{ch}\rangle$:

$$CP_{in}|f\rangle = C|f\rangle, \qquad CP_{in}|f^{ch}\rangle = P_{in}|f^{ch}\rangle \tag{2.24}$$

($CP_{in} = -P_{in}C$). In the former case the fermion (antifermion) consistently behaves like a pure scalar-charge object, whereas in the latter case it behaves like a pure pseudoscalar-charge object; in *both* individual cases (and not only in the former) P mirror symmetry may be strictly respected [32]; as particularly regards the latter case, the state of a fermion (antifermion) at rest appears to be *no longer* a P eigenstate, and P also plays an *internal* role as "(pseudoscalar)-charge conjugation" (in place of C).

3. CHIRAL-GAUGE GLOBAL SYMMETRY AND PSEUDOSCALAR-CHARGE CONSERVATION

Whether in the "Dirac" internal representation (2.5) or in the "chiral" one (2.15), the free fermion–antifermion field $\Psi(x)$ covariantly obeys the generalized Dirac equation

$$i\gamma^{\mu}\partial_{\mu}\Psi(x) = \Psi(x)M \tag{3.1}$$

M is the one-particle proper-mass operator defined by (2.1). A comparison of (2.1) with (2.14) enables one to recast this equation in the more convenient form

$$i\gamma^{\mu}\partial_{\mu}\Psi(x) = |m|\Psi^{(P_{in})}(x) \tag{3.2}$$

where P_{in} is the "intrinsic parity" operator defined by (2.14) and

$$\Psi^{(P_{in})}(x) \equiv \Psi(x)P_{in} = \psi_f(x)\langle f| - \psi_{\bar{f}}(x)\langle \bar{f}| \tag{3.3}$$

The field equation (3.2) (as well as its P_{in}-conjugated counterpart) is derivable from the real free Lagrangian density

$$\mathscr{L}(\Psi, \Psi^{(P_{in})}, \overline{\Psi}, \overline{\Psi}^{(P_{in})}, \ldots; |m|) = \tfrac{1}{4}[i(\overline{\Psi}\gamma^{\mu}\partial_{\mu}\Psi + \overline{\Psi}^{(P_{in})}\gamma^{\mu}\partial_{\mu}\Psi^{(P_{in})}) + \text{H.c.}] - \tfrac{1}{2}|m|(\overline{\Psi}\Psi^{(P_{in})} + \overline{\Psi}^{(P_{in})}\Psi) \tag{3.4}$$

where $\overline{\Psi}^{(P_{in})} = P_{in}\,\overline{\Psi}$. Regardless of the internal representation of the fields (which may even be the *chiral* one), this Lagrangian density is both manifestly P_{in}-invariant,

$$\mathscr{L}^{(P_{in})} \equiv P_{in}\mathscr{L}P_{in}^{\dagger} = \mathscr{L} \tag{3.5}$$

and P-invariant, with P acting as usual:

$$P: \quad \partial_{\mu} \to \partial^{\mu}, \qquad \gamma_{\mu} \to \gamma^0\gamma^{\mu}\gamma^0 \tag{3.6}$$

It is not, however, manifestly C-invariant, C being the *covariant* operator that acts as scalar-charge conjugation according to (2.3): for such a purpose, one should rather build an overall Lagrangian density like $\frac{1}{2}[\mathscr{L} + \mathscr{L}^{(C)}]$, with $\mathscr{L}^{(C)} \equiv C\mathscr{L}C^{\dagger}$. Yet, we may always assume that $\mathscr{L}^{(C)} = \mathscr{L}$ by imposing *ab initio* the γ-matrix representation for C:

$$\begin{aligned} \Psi C &= \gamma^5\Psi, \qquad C\overline{\Psi} = -\overline{\Psi}\gamma^5; \\ \Psi^{(P_{in})}\,C &= -\gamma^5\Psi^{(P_{in})}, \qquad C\overline{\Psi}^{(P_{in})} = \overline{\Psi}^{(P_{in})}\gamma^5 \end{aligned} \tag{3.7}$$

In (3.7), the π phase difference on passing from ΨC ($C\overline{\Psi}$) to $\Psi^{(P_{in})}C$ ($C\overline{\Psi}^{(P_{in})}$) is due to the fact that $CP_{in} = -P_{in}C$; it is by virtue of this phase difference that even the *mass* term in $\mathscr{L}$ may be left unvaried by $\mathscr{L} \to C\mathscr{L}C^{\dagger}$. On the other hand, as can be concluded from (2.12), (2.14), (2.18), and (2.19), the scalar- and pseudoscalar-charge one-particle operators Q and Q^{ch} acting in the fermion–antifermion internal space $\mathscr{S}_{in}$ may be formally written as

$$Q = qP_{in}, \qquad Q^{ch} = -q^{ch}C \tag{3.8}$$

where q and q^{ch} denote the *given* Q and Q^{ch} (nonzero) eigenvalues associated with the fermion internal states $|f\rangle$ and $|f^{ch}\rangle$, respectively. By use of (3.5), (3.7), and (3.8), it is then easy to verify that $\mathscr{L}$ has indeed the peculiar feature of being left invariant by *two individual varieties* of global $U(1)$ transformations: the former is

$$\mathscr{L} \to \exp[-i\alpha Q]\mathscr{L}\exp[i\alpha Q] \tag{3.9}$$

and the latter is

$$\mathcal{L} \to \exp[-i\beta Q^{ch}]\mathcal{L}\exp[i\beta Q^{ch}] \tag{3.10}$$

($\hbar = c = 1$), where α and β are two, scalar and pseudoscalar, (constant) real phases. As particularly regards transformation (3.10), it amounts to the *chiral gauge*

$$\begin{aligned} \Psi &\to \exp[-i\beta q^{ch}\gamma^5]\Psi, \qquad & \overline{\Psi} &\to \overline{\Psi}\exp[i\beta q^{ch}\gamma^5] \\ \Psi^{(P_{in})} &\to \exp[i\beta q^{ch}\gamma^5]\Psi^{(P_{in})}, \qquad & \overline{\Psi}^{(P_{in})} &\to \overline{\Psi}^{(P_{in}]}\exp[-i\beta q^{ch}\gamma^5] \end{aligned} \tag{3.11}$$

The conserved free currents that can independently be obtained from $\mathcal{L}$ invariance under either (3.9) or (3.10) are

$$\mathcal{J}^{(Q)} = QJ = JQ, \qquad \mathcal{J}^{(Q^{ch})} = Q^{ch}J = JQ^{ch} \tag{3.12}$$

where

$$J \equiv J^{\mu} = \tfrac{1}{2}[\overline{\Psi}\gamma^{\mu}\Psi + \overline{\Psi}^{(P_{in})}\gamma^{\mu}\Psi^{(P_{in})}] \qquad (\partial_{\mu}J^{\mu} = 0) \tag{3.13}$$

With the help of (3.7) and (3.8), it is easy to check that $\mathcal{J}^{(Q^{ch})}$ coincides with the conserved chiral current

$$J^{(\gamma^5)} \equiv \tfrac{1}{2}q^{ch}[-\overline{\Psi}\gamma^{\mu}\gamma^5\Psi + \overline{\Psi}^{(P_{in})}\gamma^{\mu}\gamma^5\Psi^{(P_{in})}] \tag{3.14}$$

which can be derived from $\mathcal{L}$ invariance under (3.11). Owing to (2.18) and (2.19), currents (3.12) are consistently such that

$$\begin{aligned} C\mathcal{J}^{(Q)} &= -\mathcal{J}^{(Q)}C, \qquad & P_{in}\,\mathcal{J}^{(Q^{ch})} &= -\mathcal{J}^{(Q^{ch})}P_{in} \\ P_{in}\mathcal{J}^{(Q)} &= \mathcal{J}^{(Q)}P_{in}\quad, \qquad & C\mathcal{J}^{(Q^{ch})} &= \mathcal{J}^{(Q^{ch})}C \end{aligned} \tag{3.15}$$

(note that J commutes with both P_{in} and C). As an improvement over the ordinary quantum field formalism, one thus may strictly deal with the conservation of both a *scalar* and a *pseudoscalar* variety of charges, which is an essential preliminary outcome for a quantum description of monopoles. The two current operators (3.12) act in the generalized Fock space (2.2) and, due to (2.20), *anticommute*. The "bare" current operator J, common to them both, is properly acting in $\mathcal{F}^o$ and only trivially acting in $\mathcal{S}_{in}$. This can be seen, e.g., if use is made of the closure relation $|f\rangle\langle f| + |\bar{f}\rangle\langle\bar{f}| = 1$ (where 1 simply denotes the identity operator in $\mathcal{S}_{in}$): with the help of it, one can recast J^{μ} in the reduced form

$$J^{\mu} = \overline{\psi}_f\gamma^{\mu}\psi_f = \overline{\psi}_{\bar{f}}\gamma^{\mu}\psi_{\bar{f}} \tag{3.16}$$

where $\psi_{\bar{f}}(\overline{\psi}_{\bar{f}})$ is the proper-mass conjugated counterpart of $\psi_f(\overline{\psi}_f)$ according to (2.8). As for the scalar-charge current $\mathcal{J}^{(Q)}$, one has

$$\mathcal{J}^{(Q)} \equiv \mathcal{J}^{(Q)\mu} = qJ^{\mu}(\mathcal{P}_f - \mathcal{P}_{\bar{f}}) \tag{3.17}$$

with

$$\mathcal{P}_f \equiv |f\rangle\langle f|, \qquad \mathcal{P}_{\bar{f}} \equiv |\bar{f}\rangle\langle\bar{f}| \tag{3.18}$$

(q being the Q eigenvalue relevant to $|f\rangle$). This shows that $\mathcal{J}^{(Q)}$, strictly speaking, is a *double*-structured current, which (in Dirac's language) provides for *both* equivalent cases when one is choosing either "particle" = fermion (and "hole" = antifermion) or "particle" = antifermion (and "hole" = fermion): the former case is selected by $\mathcal{P}_f$ and the latter by $\mathcal{P}_{\bar{f}}$. If, for example, we pick up the mere "fermionic" covariant current term $qJ\mathcal{P}_f$ and apply normal ordering to J, we are already led to a *complete* "particle + hole" current like

$$q(:J:)\mathcal{P}_f = \tfrac{1}{2}q[J - J^{(h)}]\mathcal{P}_f \tag{3.19}$$

where, as usual, $J^{(h)}$ is obtained from J by the annihilation- and creation-operator substitutions $a \rightarrow a^{\mathrm{h}}$, $a^{\mathrm{h}\dagger} \rightarrow a^{\dagger}$. Similar remarks apply to the pseudoscalar-charge current $\mathcal{J}^{(Q^{\mathrm{ch}})}$, the only difference being that the "fermionic" and "antifermionic" covariant projection operators are therein $|f^{\mathrm{ch}}\rangle\langle f^{\mathrm{ch}}|$ and $|\bar{f}^{\mathrm{ch}}\rangle\langle\bar{f}^{\mathrm{ch}}|$, respectively.

4. TWO SINGLE—"ELECTRIC" AND "MAGNETIC"—LOCAL GAUGE COUPLINGS, WITH NO POSSIBILITY OF INTERFERENCE

What has just been generally said for scalar and pseudoscalar conserved charges can particularly apply to electric and magnetic charges. Let $Q = Q_{\mathrm{e}}$ and $Q^{\mathrm{ch}} = Q_{\mathrm{m}}$ be the respective one-particle charge operators that should primarily represent them in the fermion–antifermion internal space $\mathcal{S}_{\mathrm{in}}$. These, in view of (2.20), are two *anticommuting* operators, subject to conditions (2.18) and (2.19). The same holds for their conserved free currents $\mathcal{J}^{(Q)} = \mathcal{J}_{\mathrm{e}}$ and $\mathcal{J}^{(Q^{\mathrm{ch}})} = \mathcal{J}_{\mathrm{m}}$, which are built from the "bare" current operator (3.13) according to the formulas

$$\mathcal{J}_{\mathrm{e}} = Q_{\mathrm{e}}J = JQ_{\mathrm{e}}, \qquad \mathcal{J}_{\mathrm{m}} = Q_{\mathrm{m}}J = JQ_{\mathrm{m}} \tag{4.1}$$

and obey the anticommutation rule

$$\mathcal{J}_{\mathrm{e}}\mathcal{J}_{\mathrm{m}} + \mathcal{J}_{\mathrm{m}}\mathcal{J}_{\mathrm{e}} = 0 \tag{4.2}$$

besides fulfilling the requirements (3.15). As a consequence of (2.22) and (2.23), we further have

$$\langle f|\mathcal{J}_{\mathrm{m}}|f\rangle = \langle\bar{f}|\mathcal{J}_{\mathrm{m}}|\bar{f}\rangle = 0 \tag{4.3}$$

as well as

$$\langle f^{\mathrm{ch}}|\mathcal{J}_{\mathrm{e}}|f^{\mathrm{ch}}\rangle = \langle \bar{f}^{\mathrm{ch}}]\mathcal{J}_{\mathrm{e}}|\bar{f}^{\mathrm{ch}}\rangle = 0 \tag{4.4}$$

where the two $\mathcal{S}_{\mathrm{in}}$ bases ($|f\rangle$, $|\bar{f}\rangle$) and ($|f^{\mathrm{ch}}\rangle$, $|\bar{f}^{\mathrm{ch}}\rangle$) are those diagonalizing Q_{e} and Q_{m}, respectively.

Electric and magnetic four-currents would thus be prevented at the quantum level from *locally coexisting* and making up a true *unified* electromagnetic four-current: we might at most speak of two *separate* four-currents, each one generating *its own* electromagnetic field. This would in particular account for the "absence" of magnetic sources in the customary Maxwell field equations and for the corresponding existence of a seemingly "incomplete" electromagnetic phenomenology.

According to the quantum field formalism seen in the previous section, the individual laws of electric and magnetic charge conservation should be also associated with *two distinct* global gauge symmetries, which are those under the single varieties of $U(1)$ transformations (3.9) and (3.10), with generators $Q = Q_{\mathrm{e}}$ and $Q^{\mathrm{ch}} = Q_{\mathrm{m}}$. As far as magnetic charge conservation is concerned, the related gauge symmetry would be actually equivalent to invariance under a *chiral gauge* like (3.11), with q^{ch} denoting the strength of a fermionic elementary monopole. Either of these gauge symmetries is *individually* allowed to undergo a local extension. So, on substituting $\alpha = \alpha(x)$ ($x \equiv x^{\mu}$) in (3.9), invariance can obviously be restored by adding to the free Lagrangian density (3.4) a minimal coupling term of the type

$$\mathcal{L}_{\mathrm{e}} = -\mathcal{J}_{\mathrm{e}}A \equiv -\mathcal{J}_{\mathrm{e}}{}^{\mu}A_{\mu} \tag{4.5}$$

where A_{μ} is the ordinary photon gauge field, subject to the complementary gauge transformation

$$A_{\mu} \to A_{\mu} - \partial_{\mu}\alpha(x) \tag{4.6}$$

Similarly, if $\beta = \beta(x)$ is substituted in (3.10), then invariance can be restored by adding, as a counterpart of (4.5), a minimal coupling term of the type

$$\mathcal{L}_{\mathrm{m}} = -\mathcal{J}_{\mathrm{m}}B \equiv -\mathcal{J}_{\mathrm{m}}{}^{\mu}B_{\mu} \tag{4.7}$$

where B_{μ} is a *further* (massless) gauge field, subject to the complementary gauge transformation

$$B_{\mu} \to B_{\mu} - \partial_{\mu}\beta(x) \tag{4.8}$$

Of course, as $\mathcal{J}_{\mathrm{e}}$ and $\mathcal{J}_{\mathrm{m}}$ behave like a vector and an axial-vector in space-time, so must respectively do the fields A and B to ensure Lorentz invariance of (4.5) and (4.7).

A scheme with two distinct electromagnetic four-potentials is not a new one in the literature [9,10,13,16]. The novelty is that they also enter into a pair of *independent* local gauge couplings whose interference is strictly prevented by the constraints (4.3) and (4.4). We should then be actually faced with *two, quite disjoined varieties* of electromagnetism, which may, however, be said to be reciprocally "dual" in the sense that they can be transformed into each other by the duality rotation

$$\mathcal{J}_{\mathrm{e}} \to \mathcal{J}_{\mathrm{m}}, \qquad \mathcal{J}_{\mathrm{m}} \to -\mathcal{J}_{\mathrm{e}}; \qquad A \to B, \qquad B \to -A \tag{4.9}$$

These varieties should clearly include the two independent (and dual) sets of Maxwell equations

$$\partial_\mu F_{\mathrm{e}}{}^{\mu\nu} = -\mathcal{J}_{\mathrm{e}}{}^{\nu}, \quad \partial_\mu \tilde{F}_{\mathrm{e}}{}^{\mu\nu} = 0; \qquad \partial_\mu F_{\mathrm{m}}{}^{\mu\nu} = -\mathcal{J}_{\mathrm{m}}{}^{\nu}, \quad \partial_\mu \tilde{F}_{\mathrm{m}}{}^{\mu\nu} = 0 \tag{4.10}$$

where

$$F_{\mathrm{e}}{}^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu, \qquad F_{\mathrm{m}}{}^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu \tag{4.11}$$

and

$$\tilde{F}_{\mathrm{e}}{}^{\mu\nu} = \frac{1}{2} e^{\mu\nu\rho\sigma} F_{\mathrm{e}\rho\sigma}, \qquad \tilde{F}_{\mathrm{m}}{}^{\mu\nu} = -\frac{1}{2} e^{\mu\nu\rho\sigma} F_{\mathrm{m}\rho\sigma} \tag{4.12}$$

($e^{\mu\nu\rho\sigma}$ being the completely antisymmetric unit tensor of fourth rank with $e^{0123} = 1$). Under (4.9), one in particular obtains

$$\begin{aligned} &F_{\mathrm{e}}{}^{\mu\nu} \to F_{\mathrm{m}}{}^{\mu\nu}, \quad F_{\mathrm{m}}{}^{\mu\nu} \to -F_{\mathrm{e}}{}^{\mu\nu}; \\ &\tilde{F}_{\mathrm{e}}{}^{\mu\nu} \to -\tilde{F}_{\mathrm{m}}{}^{\mu\nu}, \quad \tilde{F}_{\mathrm{m}}{}^{\mu\nu} \to \tilde{F}_{\mathrm{e}}{}^{\mu\nu} \end{aligned} \tag{4.13}$$

Two such separate sets of Maxwell equations can be merged to give the *symmetric* set of field equations

$$\partial_\mu F^{\mu\nu} = -\mathcal{J}_{\mathrm{e}}{}^{\nu}, \qquad \partial_\mu \tilde{F}^{\mu\nu} = -\mathcal{J}_{\mathrm{m}}{}^{\nu} \tag{4.14}$$

where $F^{\mu\nu}$ and $\tilde{F}^{\mu\nu}$ are formally defined as

$$F^{\mu\nu} = F_{\mathrm{e}}{}^{\mu\nu} + \tilde{F}_{\mathrm{m}}{}^{\mu\nu}, \qquad \tilde{F}^{\mu\nu} = \tilde{F}_{\mathrm{e}}{}^{\mu\nu} + F_{\mathrm{m}}{}^{\mu\nu} \tag{4.15}$$

and $\tilde{F}^{\mu\nu} = \frac{1}{2} e^{\mu\nu\rho\sigma} F_{\rho\sigma}$. As can be easily checked, transformations (4.13) actually correspond to the overall rotation

$$F^{\mu\nu} \to \tilde{F}^{\mu\nu}, \qquad \tilde{F}^{\mu\nu} \to -F^{\mu\nu} \tag{4.16}$$

so that the equation set (4.14) is left *unchanged* by (4.9). Thanks to the anticommutivity quantum property of the electric and magnetic currents in (4.14), a description invariant under (4.9) may thus be attained which is also *free* from singularity problems.

5. A MAGNETIC ELEMENTARY CHARGE "DUAL" TO THE ELECTRIC ONE

In the foregoing, no particular link like the Dirac quantization condition has emerged for electric and magnetic charge eigenvalues. This is closely related to the anticommutivity property of the corresponding one-particle charge operators Q_e and Q_m, which should indeed prevent any *mutual* interaction of electric and magnetic charge eigenstates: two *separate* local gauge couplings unable to interfere can but be generated by Q_e and Q_m. Thus, one might even argue that building the unified field equations (4.14) from the two dual sets of Maxwell equations (4.10) is just a formal matter devoid of any clear physical motivation.

Despite that, a true (and very intimate) connection between the couplings (4.5) and (4.7) may all the same be conjectured with the help of Dirac's fermion theory. One should only allow for the fact that an electrically charged Dirac spin-1/2 fermion naturally carries also a magnetic dipole moment, and so it cannot trivially stand for an *eigenstate* of magnetic charge with a null eigenvalue. As is well known, such a dipole moment is directly related to the magnitude, say $|e|$, of the electric charge carried by the fermion: it amounts to that of two (opposite) magnetic point charges of magnitude $|e|/2$ placed at a relative distance equal to the effective fermionic Compton wavelength (resulting from radiative corrections). If we want to interpret this outcome according to the new quantum views above, we have first to consider that, in line with (2.22), an electric charge eigenstate such as the one representing a charged Dirac fermion might only exhibit a *vanishing* expectation value of magnetic charge. Confining ourselves to charged leptons, we may put $|f\rangle = |-e\rangle$ in (2.22), where $|-e\rangle$ is an eigenstate of the electric charge one-particle operator Q_e with eigenvalue $-e$. As the trivial case of a null magnetic charge *eigenvalue* associated with $|-e\rangle$ seems to be excluded by the presence of the magnetic dipole moment, the only possibility left is to write [13]

$$|-e\rangle = 2^{-1/2}(|\mathrm{g}\rangle + |-\mathrm{g}\rangle), \qquad \langle -e|Q_m|-e\rangle = 0 \tag{5.1}$$

where $|\mathrm{g}\rangle$ and $|-\mathrm{g}\rangle$ are eigenstates of the one-particle magnetic charge operator Q_m with eigenvalues g and $-$g. A Dirac charged lepton may thus be unconventionally viewed as also bearing a *single* magnetic point charge of squared magnitude g^2 subjected to *a maximal uncertainty in sign*. Moreover, since the internal state $|-e\rangle$ is a parity eigenstate, it is evident that the two (equally probable) signs of this magnetic charge should also correspond to *two alternate locations of it which are spatially inverted images of each other.* These (equally probable) locations of the magnetic charge would fall within the minimum range of localization given by the effective Compton wavelength of the lepton; they would only contribute alternately to the dipole moment, and two such alternate contributions would further be *equivalent*. So, after all, the single magnetic charge being conjectured should *always* yield (regardless of sign and corresponding location) a dipole moment which can be evaluated in magnitude as the product of $|\mathrm{g}|$ by *half* the Compton wavelength involved. This enables one *to set*

$$\mathrm{g}^2 = e^2 \tag{5.2}$$

and state that *the magnetic dipole moment of a charged lepton, seemingly due to an indivisible pair of opposite poles of magnitude* $|e|/2$, *actually results from the internal presence of one magnetic point charge of absolute strength* $|e|$ *which is bound to exhibit a maximal uncertainty in sign*. The idea of such a charge "dual" to the electric one clearly comes up to the natural expectation for a *single* electromagnetic coupling constant [33]; it also gives a sound physical motivation for a *unified* scheme based on the symmetric field equations (4.14).

6. CONJECTURE OF "DUALITY EQUIVALENCE"

The basic change brought in by the present approach to dealing with the monopole problem is that an electric and a magnetic point charge are now to be represented as two (scalar and pseudoscalar) *one-particle quantum operators* like Q_e and Q_m. These are *anticommuting* charge operators, such that, for a Q_e eigenstate with eigenvalue $q_e \neq 0$, one has

$$\langle Q_e \rangle = q_e \neq 0 \Rightarrow \langle Q_m \rangle = 0 \tag{6.1}$$

According to (6.1), a particle that is revealing itself as an electric charge eigenstate with a nonzero eigenvalue cannot simultaneously look like a magnetic charge eigenstate with a nonzero eigenvalue: it is strictly bound to display a *null* expectation value of magnetic charge. Within the unified fermion–antifermion quantum field formalism used here, the two charges Q_e and Q_m are not linked by any Dirac-like quantization condition; they may only generate *quite distinct* local gauge couplings, which can be formally merged into an "overall" coupling given by the whole term

$$\mathscr{L}_{em} = \mathscr{L}_e + \mathscr{L}_m = -(Q_e J^\mu A_\mu + Q_m J^\mu B_\mu) \tag{6.2}$$

In (6.2), A_μ and B_μ are two independent (massless) gauge fields (behaving as a vector and an axial-vector, respectively) and J^μ is the fermion–antifermion current operator (3.13). Due to the anticommutivity property of Q_e and Q_m, the coupling (6.2) may at most express either a *pure electric* or a *pure magnetic* monopole interaction at a time; and it can be reduced to the *mere* coupling to the ordinary electromagnetic gauge field A_μ by taking the Q_e and Q_m expectation values in line with (6.1).

As seen in the previous section, Dirac's fermion theory (supported by experience) makes it legitimate to set, for charged leptons,

$$\langle Q_m{}^2\rangle = \langle Q_e{}^2\rangle = e^2 \tag{6.3}$$

Condition (6.1) applied to a charged lepton reads

$$\langle Q_e\rangle = -e \Rightarrow \langle Q_m\rangle = 0 \tag{6.4}$$

So, if the lepton were alternatively allowed to manifest itself as a magnetic charge eigenstate, we might *quite symmetrically* write, in view of (6.3),

$$\langle Q_m\rangle = -e \Rightarrow \langle Q_e\rangle = 0 \tag{6.5}$$

On the other hand, strictly speaking, it is just a matter of definition which of the two charges Q_e and Q_m in (6.2) is the scalar or pseudoscalar one and which of the associated gauge fields A_μ and B_μ is accordingly the vector or axial-vector one: what only matters, in either case, is that Q_e and Q_m (taken altogether) transform *differently* under parity [14] due to their being two *anticommuting* charges. Hence, the supposed *alternative* behavior of a charged lepton as a magnetic monopole of strength $-e$ should indeed turn out to be *indistinguishable* from its ordinary behavior as an electric monopole. The strict assumption (6.3) therefore involves a "dual" model of a charged lepton such that the it may be thought of like *either an electric monopole*, endowed with a *magnetic* dipole moment in line with (6.4). *or a magnetic monopole*, endowed with an *electric* dipole moment in line with (6.5). Such a *maximum* degree of symmetry, which we may call "duality equivalence," is to be properly expressed by use of our unified fermion–antifermion quantum field formalism (see Section 2): it implies that the "Dirac" internal state basis ($|f\rangle$, $|\bar{f}\rangle$) diagonalizing Q_e and the "chiral" one ($|f^{ch}\rangle$, $|\bar{f}^{ch}\rangle$) diagonalizing Q_m can both *equivalently* represent the same charged lepton and related antilepton. Parity symmetry, in either picture, is fully respected, even though in a diametrically opposed way: as particularly regards the latter picture, the two (fermion and antifermion) Q_m eigenstates $|f^{ch}\rangle$ and $|\bar{f}^{ch}\rangle$, instead of being parity eigenstates, are on the contrary *interchanged* by parity [32]. In this connection, it should be emphasized that the *electric* dipole moment associated with the magnetic monopole would involve *no* actual breaking of either parity or time-reversal symmetry, as the two internal states $|f^{ch}\rangle$ and $|\bar{f}^{ch}\rangle$ are not required at all to be individually symmetric under parity.

The conjecture of duality equivalence, marked (in the lepton case) by the *universal* (fine structure) coupling constant α, leads to a great simplification of the monopole problem: on one hand, we are strictly faced with a *completely symmetric* set of Maxwell equations like (4.14), but, on the other hand, no *new* electromagnetic phenomenology is to be expected as a "complement" of the one to which we are accustomed. In such a framework of maximal symmetry, it is just the quantum property of *anticommutivity* between electric and magnetic charges that accounts for a seemingly “incomplete”, well-established phenomenology: as a consequence, the single “electric” and “magnetic” fermionic monopoles may be said to be “complementary” to each other only in the sense that they provide two equivalent *alternative* images of one and the same “electromagnetic” monopole. Of course, this pure quantum view is indissolubly connected with the strict physical condition (6.3); so it has nothing to do [16] with the well-known classical formal arguments against the magnetic monopole conjecture [34–36] (which apply only to charges treated as mere *c*-numbers).

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